IFORS' Operational Research Hall of Fame
Kenneth J. Arrow

Significant contributor to the theory of social choice and economic equilibria, and many fields in the decision sciences, including decision theory, dynamic programming, inventory and production theory, linear and non-linear programming, and economics of medical care.

Born: 23 August 1921, New York, NY, USA.

Education: B.S. Social Science, City College of New York (1940), M.A. Mathematics (1941) and Ph.D. Economics (1951), Columbia University.

Currently: Joan Kenney Professor of Economics Emeritus and Professor of Operations Research Emeritus, 1991.

Key positions: Assistant Professor of Economics, University of Chicago (1948–1949); Consultant, RAND Corporation (1949); Acting Assistant Professor (1949), Associate Professor of Economics and Statistics, Stanford University (1950–1953); Professor of Economics, Statistics, and Operations Research, Stanford University (1953–1968); Professor of Economics, Harvard University (1968–1974); James Bryant Conant University Professor, Harvard University (1974–1979) and Joan Kenney Professor of Economics and Professor of Operations Research, Stanford University (1979–91).

Awards and Recognition: John Bates Clark Medal, American Economic Association (1957); Nobel Prize for Economics (1972); John von Neumann Theory Prize, TIMS/ORSA (1986); Fellow, Center for Advanced Study in the Behavioral Sciences (1956–1957); Fellow, Churchill College, Cambridge (1963–1964, 1970, 1973, 1986); Guest Professor, Institute for Advanced Studies, Vienna (1964, 1970); Visiting Fellow, All Souls College, Oxford (1996); Member, National Academy of Sciences and the American Philosophical Society; Fellow, INFORMS, American Academy of Arts and Sciences, American Economic Association, Econometric Society, Institute of Mathematical Statistics, American Association for the Advancement of Science, American Finance Association and American Statistical Association; more than 20 honorary degrees; President, Econometric Society (1956); The Institute of Management Sciences (1963); American Economic Association (1973); Western Economic Association (1981); International Society for Inventory Research (1982–1988); International Economic Association (1983–1986); Society for Social Choice and Welfare (1992–1993); Honorary President, Scientific Council, Institut du Développement Durable et des Relations Internationales (2002–).

One of the most prominent economic theorists of the 20th century, Kenneth J. Arrow has made fundamental contributions to numerous fields, most of them concentrated around general equilibrium theory and welfare economics, of which he can be considered one of the primary architects. He was awarded the Nobel Prize for Economics in 1972, for ''pioneering contributions to general economic equilibrium theory and welfare theory''.

Kenneth Arrow was a thorough New York City product: born and raised in the city, educated at City College (CCNY), where he received the Gold Pell medal for the highest grades, and subsequently a graduate student in mathematical statistics at Columbia under Harold Hotelling and Abraham Wald. It was at Columbia that he was nudged towards economics. Finishing his Ph.D. coursework in 1942, Arrow's subsequent dissertation was to become a decade-long affair. After a stint during World War II in the Weather Division of the USAF, Arrow returned to Columbia in 1946 and, with dissertation topic still missing, began taking steps to move into the private sector – passing a series of actuarial examinations and searching for jobs in the insurance industry. At this time, Arrow attended Jacob Marschak's mathematical seminar at the New School. Hotelling and Wald, concerned by Arrow's lack of progress on his dissertation, convinced him to join Marschak at the Cowles Commission in Chicago as a research associate in April 1947. Almost immediately he met a graduate student, Selma Schweitzer, and within a few months they were married. The marriage produced two sons, David and Andrew, and, as he himself put it in 2002, Selma and he ''are still enjoying our happiness''. His first non-weather paper was on sequential decision problems, co-authored with Blackwell and Girshik (1949).

At the Cowles Commission, Arrow absorbed much of the research programme being set up under Marschak and Koopmans. Arrow contributed a piece to the famous Koopmans-edited Cowles monograph on activity analysis (1951) and wrote an article with Marschak and Harris on inventory policy (1951). Marschak and Arrow realized that there were two types of inventory model: one dynamic but deterministic, the other stochastic but static. They saw that these two could be combined, and considered solutions of the (S, s) form (order only when the stock on hand falls to s and then order $(S–s)$ to raise the stock to S). The behavior of the stock was a complicated Markov process, and they turned to Harris for help. He derived the distribution of the processes and thus the expected discounted sum of costs for a fixed (S, s) policy. Their paper stimulated a large literature. They had found the best policy among this limited class of policies, but were there better policies of a different form? The matter was resolved in what Arrow calls ''a brilliant paper'' by Scarf (1960), who demonstrated that under a broad assumption about the loss function, the (S, s) policy was indeed optimal. Scarf had met Arrow at RAND, and subsequently been invited to spend the academic year 1956–1957 at Stanford University, an invitation that was subsequently extended. He published on inventory and production (1958) with Arrow and Karlin. Arrow's interest in inventory problems continued throughout his career, and in 1982 he became the first President of the International Society for Inventory Research.

After two highly inspiring years at Cowles, Arrow moved for a summer to the RAND Corporation, before moving to teach at Stanford – where he has remained until today, save for an interlude at Harvard from 1968 to 1979. For a number of summers he continued to visit RAND. One of the research items at RAND was the then novel use of game theory to analyze international conflicts and strategy. However, game theory itself presupposed the parties possessed some sort of utility functions. For parlor games with individual people this is acceptable, but when dealing with nations as a whole involved in strategic affairs the question of

"America's" utility and "Soviet Union's" utility was an altogether different issue. Thus, Arrow wondered, under what conditions might it be reasonable to assume that collectivities such as nations possessed nicely behaved utility functions? At long last, a dissertation topic was found: subsequently published as the momentous classic, *Social Choice and Individual Values* (1951).

At the heart of Arrow's (1951) dissertation was the presumption that social choice orderings could be derived from individual choice orderings in a simple axiomatic manner. Arrow's astounding conclusion, since then referred to as the "Arrow Impossibility Theorem", was that a certain set of quite acceptable axioms on social choice orderings necessarily implied that there would be a "dictator" (i.e. that a single agent's own preferences over outcomes would dominate everybody else's). This celebrated result has since created a research industry in social welfare theory to which Arrow himself has made further significant contributions.

Arrow's dissertation was concurrently published with another formidable piece of Cowles-inspired work: his proof of the First and Second Welfare theorems in general equilibrium theory (1951). Gerard Debreu, then at Cowles, had independently proved the same theorem. Arrow and Debreu began their famous collaboration that culminated in the celebrated "Arrow and Debreu" proof of existence of a competitive equilibrium (1954).

All the while, Arrow had been groping for incorporating uncertainty into a general equilibrium context. In a path-breaking article, Arrow (1953) suggested that a simple procedure would be to consider markets for "state-contingent" commodities. Consequently, he demonstrated that in equilibrium with a full set of state-contingent markets, there would be an optimal allocation of risk. However, Arrow noted that a full set of state-contingent commodities might seem too unrealistic. In the same article, Arrow proposed the famous contrivance of what became known as "Arrow securities", i.e. securities that paid a unit of account in a particular state and nothing otherwise. Arrow demonstrated that a full set of state-contingent commodity markets could be replaced by a considerably smaller set of Arrow securities that spanned the various possible states, and that, consequently, the optimal allocation of risk would be identical as in an Arrow–Debreu model with a full set of state-contingent markets.

Arrow subsequently turned his attention to a new topic: the issue of "stability" of competitive equilibrium with multiple markets. His interest in this topic was initiated by his work on "D-stability" with McManus (1958) that was followed up by a famous article with Hurwicz (1958) on local stability. Perhaps his most famous achievement in this regard was to provide sufficient conditions (i.e. WARP) for global stability of equilibrium in an article with Block and Hurwicz (1959). His subsequent extensions and clarifications with Hurwicz (1960, 1962) outlined both the achievements and problems of stability theory.

Arrow had, in the meantime, maintained his interest in mathematical programming. With Hurwicz and Uzawa he had published a text on linear and non-linear programming (1958), and in 1961, with them, he produced a famous rather weak constraint qualification (that replaced Kuhn–Tucker's) for obtaining the saddle-point characterization of a local solution to a non-linear programming problem. With Enthoven (1961) he produced several famous results in characterizing optimization problems when both the maximand and constraint functions are quasi-concave.

He moved concurrently into the issues of production and growth. In 1961, with Hollis Chenery, B.S. Minhas and Robert Solow, Arrow introduced the famous "Constant Elasticity of Substitution" (CES) production function. In 1962, he produced two famous pieces on "learning-by-doing", the predecessor of modern endogenous growth theory. His landmark

contributions continued: putting his early experience with actuarial work to good use, Arrow produced his famous 1963 paper on medical insurance that introduced the concept of moral hazard into economics and announced the dawn of information theory. His 1965 lectures *Aspects of the Theory of Risk Bearing* introduced the ''Arrow-Pratt'' measure of risk-aversion as well as popularizing asymmetric information and the terms ''moral hazard'' and ''adverse selection''. Much of Arrow's work on moral hazard, optimal insurance and optimal risk-bearing allocations was collected in his 1971 book, *Essays in the Theory of Risk-Bearing*. Arrow's contributions to education and racial discrimination (1972, 1973) have since become well-known classroom applications of signalling and screening mechanisms under conditions of asymmetric information.

Arrow had linked the theory of public investment with uncertainty in a famous article with Lind (1970) arguing for the risk-bearing role of government. His concern with mathematical programming and public policy led him naturally to the topic of optimal policy – in particular, to the use of optimal control theory as a guide to resource allocation, inventory policy, public investment, etc. In collaboration with Mordecai Kurz, that culminated in their well-known 1970 book, Arrow presented numerous applications and extensions of the, then, rare use of Hamiltonians. Arrow's ''sufficiency'' conditions for an optimum generalized the Mangasarian conditions.

In 1971, Kenneth Arrow and Frank H. Hahn produced their treatise/textbook *General Competitive Analysis* that remained, until recently, a definitive treatment of Walrasian general equilibrium theory. Although research in general equilibrium has since moved in very different directions than those considered by Arrow and Hahn, it was at least partly because of their critical assessments of the treatment of money, uncertainty and stability in a G.E. context that led economists to recast these issues in a different light.

Although Arrow has produced more than a lion's share of groundbreaking contributions to general equilibrium theory, social welfare theory, growth, production, uncertainty, information and optimal public policy, he has not rested on his laurels and continues to produce apace. For instance, his 1979 work with Radner on the theory of ''teams'' and with Chang on the theory of natural resources (1980) has opened new avenues in the theory of organization and resource allocation. Even more recently, Arrow has teamed up with Hahn again to begin tackling the issue of ''endogenous uncertainty'' (1999).

Arrow was the 10th President of the Institute of Management Sciences in 1963, which in 1986, in conjunction with the Operations Research Society of America, awarded him the prestigious John von Neumann Theory Prize. The citation states that it was awarded for ''landmark contributions to the theory of social choice and economic equilibria, and for fundamental and prodigious contributions to an astonishing array of fields in the decision sciences. These include decision theory, the theory of risk-bearing, the economics of information and organization, dynamic programming, inventory and production theory, linear and non-linear programming, advertising policy, economics of medical care, theory of job discrimination, economic growth theory, finance, price theory, maintenance policy, economics of education, natural resource policy, and technological innovation. His work exhibits extraordinary theoretical power and often provides stunning insights on the fundamental issues of the day''.

Kenneth Arrow is perhaps one of the most respected and admired living economists. In many ways, his life is exemplary in that the almost incredible successes that have accompanied him have not, in any way, hardened into that arrogance and pettiness so common among professional

scholars. By all accounts, Arrow ranks highly among economists and non-economists, orthodox and heterodox, for his scholarly depth, his wide-ranging interests, his personal and intellectual generosity and openness, and his consistent refusal to engage in ideological quibbling.

Arrow's achievements were neither the result of wild luck nor were they quickly or cheaply bought by hack work, but rather the outcome of an often painful but nonetheless continuous dedication to the task of the scholar. It is evident throughout his work that Arrow has maintained the highest standards of rigor, avoiding oversimplification and ideological rhetoric, clearly aware of and indeed actively demarcating the limits of applicability of economic theory. In doing so, he has achieved and granted to us a far deeper understanding of both economics and the economic process than might otherwise have been possible.

Still productive in his eighties, he set out his agenda for the future (2004). This included several areas of policy, starting from the least likely policy at this point of time, the use of the tax system for redistribution, through discussion of prudent macroeconomic coordination without the strings of the Maastricht Treaty in business troughs, regulation of financial markets, agricultural policies, and health issues.

Kenneth Arrow is most deserving of inclusion in the Operations Research Hall of Fame. Whilst at Stanford, he has been a professor of Operations Research since 1953, and in 1957 published his text on decision theory and OR. To quote again the citation for the von Neumann Prize, "his lightning-quick mind, his awesome wealth of knowledge, his extraordinary breadth of interests, his elegant prose and language, and his great personal warmth have inspired and charmed countless students, colleagues and associates." His powerful theoretical contributions have illuminated vast areas of management science and operations research.

Gonçalo L. Fonseca and Graham K. Rand

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