

Decision Modeling

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David M. Tulett

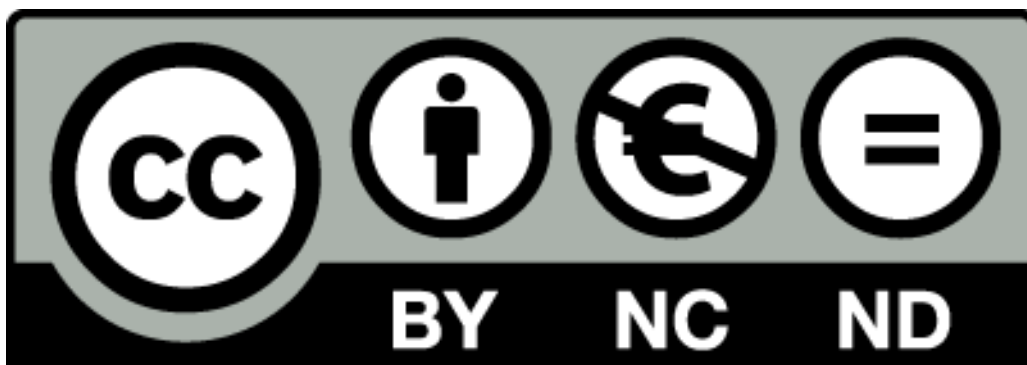
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1. This document was written using the very versatile L^AT_EX and P^STricks programs, which are open-source software. L^AT_EX is particularly good at

writing mathematical expressions, and PSTricks produces excellent graphics. Both can be downloaded for free from the T_EX Users Group (TUG) at <http://www.tug.org>.

2. A feature of Excel that is heavily used in this document is the Solver. The Solver is made by Frontline Systems, Inc., <https://www.solver.com>.
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4. The author wishes to thank retired professor Austin Redlack, Ph.D, with whom he worked on materials for courses that were developed in the period 1987-1997, and which continued to be used in courses that were offered up to 2011. In a few cases, examples from those works have been maintained in this document.
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Author

David M. Tulett obtained a B.Sc. in mining engineering and a Ph.D in management (dual major in operational research and finance) from Queen's University (Canada). He is member of the Faculty of Business Administration at Memorial University, St. John's, NL, Canada.

Preface

In this e-book, a word, phrase, or page number in red indicates a link to somewhere else in this document; a word or phrase in pink indicates a link to the web. In particular, both the Table of Contents and the Index are linked to the appropriate places in the main body of the material.

The reader needs to have learnt, or be prepared to learn, the basics of spreadsheets. A quick overview of the basics of spreadsheet operations using the syntax of Excel is provided in Chapter One. This e-book should be read in conjunction with any other materials which have been required for your course.

A great deal of care has gone into the preparation of this document, but there may well be some errors lurking somewhere. Readers who spot such errors, or who simply want to offer suggestions for improvement, are encouraged to contact the author by email.

David M. Tulett

Email: dtulett@mun.ca

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