Proceedings of Dynamic Systems and Applications 7 (2016) 1–7

Monte Carlo Simulation in Business Applications

Dr. N.S.Nilakantan

K. J. Somaiya Institute of Management Studies and Research, Vidya Vihar, Mumbai, India, 400077

ABSTRACT: In business practice, Monte Carlo (MC) simulation is extensively used for many purposes like sensitivity analysis, risk quantification and analysis, prediction etc. MC simulation creates many artificial futures for a given situation by generating number of sample paths of outcomes. The process of simulation assumes importance in the context of our inability to mathematically model the business process completely and also the flaw of averages (due to Savage). While performing MC simulation, precision and error control are important and can be achieved by appropriate sampling. Discussion will be on how best to use MC simulation in business applications with inputs from literature and the authors work.

AMS (MOS) Subject Classification. 65C05

1 INTRODUCTION

There are many stories about the advent of Monte Carlo simulation. One story credits it to a scientist Stanislav Ulam who got the idea while recuperating in his hospital bed. He took his idea to Von Neumann, who approved it and thus sowed the seeds of Monte Carlo simulation in 1946. It is now almost 70 years after that and perhaps a right time to review the progress made since then. In this talk, we discuss a few aspects of the theory behind Monte Carlo simulation, some representative simulation applications, and a few simulation enablers.

Simulation refers to recreating or creating alternate scenarios of a situation based on certain inputs. For a practitioner, simulation enables solving complex practical problems with sufficient ease. A simulation procedure usually calculates various scenarios of a model by repeatedly picking up value from either historical data or a user-defined probability distribution for the uncertain variables in the model.

2 MONTE CARLO SIMULATION

The basic building block of Monte Carlo simulation is a computerized version of a roulette wheel with many million random numbers around its edge. Monte Carlo (MC) simulation is a method of parametric simulation, where we assume specific parameters of a suitable probability distribution for the uncertain variables in the model. MC simulation was central to the simulations required for the Manhattan project, and became popular in the fields of physics and operations research. The procedure was described by David Hertz in a 1964 HBR article and popularized in financial circles by sophisticated users. Today, spreadsheet-based Monte Carlo simulation software is widely available and is being used in fields as diverse as petroleum exploration, financial engineering, defense, banking, and retirement portfolio planning. MC simulation is a viable alternative to complex stochastic closed-form mathematical models. If correctly modeled, MC simulation obtains similar answers to the more elegant mathematical models. Moreover, there are many complex real-life situations where closed-form mathematical models are not available and MC simulation is the only recourse.

2.1 Random Numbers

In general, we generate random numbers, xs, that belong to some domain, $x \in [xmin, xmax]$, in such a way that the frequency of occurrence, or probability density, will depend upon the value of x in a prescribed functional form f (x). Random numbers generated from standard uniform distribution are used in sampling from defined distributions. While true (0, 1) random numbers can be generated only by electronic devices, simulation models executed on computer use arithmetic operations and algorithms to generate pseudo- random numbers. Many programming languages have the ability to generate pseudo-random numbers which are effectively distributed according to the standard uniform distribution.

Several methods are available to transform the uniform random variate on the unit interval into another functional form. Inverse transformation, one of the most common and easy-to-understand methods, involves describing the cumulative distribution function F(x) associated with the function f(x) and generating the probability density function through the inverse function F-1(x) for any value of uniform random variate U (0, 1). If such inversion is not feasible, we use

Received September 30, 2015 Dynamic Publishers, Inc.

other methods which include composition, convolution etc, statistical methods based on sampling, and methods based on number theory. Specific methods are used to generate bivariate correlated distributions, by generating first independent distributions and providing the desired correlation by rotation of the axes. Even truncated distributions can be generated through an appropriate method which works well with the inverse transformation technique.

2.2 Probability Distributions

An assumed probability distribution or distributions is at the core of any MC simulation. Any probability distribution is characterized by a probability density function and a few defined parameters. For example, a continuous uniform distribution is characterized by the probability density function

$$f(x) = \frac{1}{b-a}\, for\, a \leq x \leq b\, and = 0\, for\, x > a\, or\, x > b. \tag{1}$$

The continuous uniform distribution is a family of symmetric probability distributions such that for each member of the family all intervals of the same length on the distributions support are equally probable. The support is defined by the two parameters a and b its minimum and maximum values. A standard uniform distribution has a =0 and b=1. The cumulative distribution function is

$$F(x) = 0\, for\, x < a, \, = \frac{x-a}{b-a}\, for\, a \leq x < b, \, = 1\, for\, x \geq b. \tag{2}$$

Its inverse is

$$F^{-1}(p) = a + p * (b-a)\, for\, 0 < p < 1. \tag{3}$$

Saucier (2000) provides a detailed exposition on generating statistical distributions in a C++ environment. This publication also highlights the overuse of the continuous normal distribution and the discrete Poisson distribution and provides the required codes for using lesser-known distributions.

2.3 Distributional Fitting

While it is possible to run the simulation trials by assuming parameters of a certain population, it is more preferable to make distributional assumptions correctly based on historical data. This is done by converting the raw data into a histogram and comparing it with that of regular distributions. Based on the assumed distribution and its parameters, the distributional fitting is performed through either a chi-square Goodness-of-fit test or a Kolmogorov-Smirnov test. While the former is used to test discrete distributions, the latter is used for continuous distributions. The chi-square and K-S test are semi-parametric and nonparametric in nature and are better suited for goodness-of-fit tests of non-normal and normal distributions. There are other distributional tests like the Anderson-Darling, Shapiro- Wilks which are very sensitive parametric tests. These may not be appropriate in MC simulation.

2.4 Chi-square test

The chi-square (CS) test is a goodness-of-fit test, which can be applied to any univariate distribution for which you can calculate the cumulative distribution function. The test requires a sufficient sample size for the CS approximation to be valid and is sensitive to the choice of bins. Also it can be applied to discrete distributions like binomial, Poisson etc. The null hypothesis that the data set follows a specified distribution is tested using the CS statistic defined as

$$\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}, \tag{4}$$

where O_i is the observed frequency for bin i and E_i is the expected frequency for bin i.The expected frequency is calculated by

$$E_i = N(F(Y_U) - F(Y_L)) \tag{5}$$

where F is the cumulative distribution function for the distribution being tested, $Y(_U)$ is the upper limit for class i, $Y(_L)$ is the lower limit for class i, and N is the sample size. The test statistic follows a CS distribution with (k-c) degrees of freedom where k is the number of nonempty cells and c = the number of estimated parameters (including location, scale, and shape parameters) for the distribution +1. For example for a two-parameter binomial distribution, c=3.The hypothesis that the data are form a population with specified distribution is rejected, if $\chi^2 > \chi^2(\alpha, k-c)$ where $\chi^2(\alpha, k-c)$ is the CS percent point function with k-c degrees of freedom and a significance level of α. A low p-value indicates a bad fit (null hypothesis rejected) while a high p-value indicates a statistically good fit.

2.5 Kolmogorov-Smirnov (K-S) test

The KolmogorovSmirnov statistic quantifies a distance between the empirical distribution function of the sample and the cumulative distribution function of the reference distribution, or between the empirical distribution functions of two samples. The test is nonparametric based on the empirical distribution function of the sample data set and the distribution of the K-S test statistic does not depend on the underlying cumulative distribution function being tested. It only applies to continuous distributions, and it tends to be more sensitive near the centre of the distribution than at the distributions tails. We also need to specify the distribution fully.

The null distribution of this statistic is calculated under the null hypothesis that the samples are drawn from the same distribution (in the two-sample case) or that the sample is drawn from the reference distribution (in the one-sample case). Given N ordered data points $Y_1, Y_2, ., Y_n$, the empirical distribution function is defined as $E_n = n(i)/N$ where n(i) is the number of points less than Y_i where Y_i values are ordered from the smallest to the largest value. The hypothesis is tested using the K-S statistic defined as

$$KS = max_{1 \leq i \leq N} \mid F(Y_i) - \frac{i}{N} \mid, \tag{6}$$

where F is the theoretical cumulative distribution of the continuous distribution being tested. This distribution must be fully specified (i.e. the location, scale, and shape parameters cannot be estimated from the data). A low p-value (e.g. less than 0.05, 0.01) leads to rejection of null hypothesis and indicates a bad fit whereas a high p-value indicates a statistically good fit.

2.6 Precision Control in Simulation

While a large number of trials are expected to produce reliable estimates with sufficient precision, we can carry out precision control by following certain statistical sampling procedures. In order to remove subjectivity in estimating the required number of trials, precision control is done using the confidence intervals to determine when a specified accuracy of a statistic has been reached. If we have a 5 percent error level with respect to the mean at the 95 percent confidence level, the number of trials required to obtain this precision is based on the following confidence interval of

$$\overline{X} \pm Z_{0.05} \frac{s}{\sqrt{n}}$$

where $Z_{0.05} \frac{s}{\sqrt{n}}$ is the error of 5 percent level, $\overline{X}$ is the sample average, Z is the standard-normal Z-score obtained from 95 percent precision level, s is the sample standard deviation and n is the number of trials required to obtain this level of error with the specified precision. Very large number of trials also involves expensive computer time, which can be saved by adopting variance reduction procedures to reduce the variance and thereby reduce the number of trials. These include antithetic variable technique, control-variate technique, importance sampling, stratified sampling, moment matching etc.

2.7 Bootstrap Simulation

An alternative to MC simulation is nonparametric bootstrap simulation in which historical data is used and no distributional parameters are assumed. In essence, bootstrap simulation is an alternative to classical hypothesis testing methods based on normal distribution of the sample statistic. Classical methods offer higher power in their tests, but relies on normality assumptions and can be used only to test the mean and variance. Bootstrap simulation, on the other hand, provides lower power but is nonparametric and distribution-free, and can be used to test any distributional statistic, like skewness etc. Nonparametric simulation uses raw data directly and hence may require additional procedures of cleaning the data (e.g. outliers and non-sensical values).

3 SIMULATION APPLICATIONS

3.1 Applications to Portfolio Optimization

Markowitz theory of portfolio selection (Markowitz, 1952) is an important contribution in the area of mathematical finance for which he was awarded the nobel prize in 1990. His theory was based on three important principles: risk, reward, and the correlation among the assets in the portfolio. While risk was measured by standard deviation, and correlation by a measure called covariance, reward was measured by the mean, specifically arithmetic mean. In the recent past, some limitations of the Markowitz model have been studied and some researchers have presented improvements incorporating alternate concepts, primarily dependent on simulation.

The importance of simulation studies can be better understood also from a consideration of the ‘Flaw of Averages’(Savage, 2009) which simply states that plans based on assumptions about average conditions usually go wrong.

Whenever we use an average to represent an uncertain quantity, we are distorting the results because an average measure ignores the impact of the inevitable but possible variations. It is also true that the average of a function of many random variables is not equal to the function of the averages of the same variables unless the function is linear. In the context of flaw of Averages, even a nonparametric bootstrap simulation is quite useful.

Though the mean-variance portfolio model of Markowitz addresses the flaw of averages, associated with the mean by distinguishing between different investments with the same average (expected) returns but with different risks (standard deviation), Kaplan and Savage (2009) say the use of standard deviation and covariance introduces a higher-order version of 'flaw of averages', in that these concepts are themselves a version of averages. They present an updated version of their model "Markowitz 2.0", addressing the several limitations of mean-variance optimization (MVO). Their updated model focuses on five specific enhancements:

1). Use of a scenario-based approach to allow fat-tailed distributions, 2). Adoption of the long-term geometric mean in place of the arithmetic mean, 3). Choice of Conditional Value at Risk (CVaR) in place of standard deviation, 4). Scenario-based modelling with Monte-Carlo simulation to accommodate any number of distributions to describe the returns and 5). Exploit new statistical technologies pioneered by Savage in the field of Probability Management.

With phenomenal speed of computers, the field of Probability Management is able to extend data management to probability distributions rather than numbers. The key component of Probability Management is the Distribution String, or $DIST^{TM}$, which can encapsulate thousands of trials as a single data element. The use of DISTs greatly saves on storage and speeds up processing time, so that a Monte Carlo simulation consisting of thousands of trials can be performed on a personal computer in an instant. Kaplan and Savage (2009) say, "while not all asset-management organizations are prepared to create the DISTs needed to drive the GM-CVaR optimization we described, some outside vendors, such as Morningstar Ibbotson, can fulfill this role."

Gulten and Ruszczvnski (2015) adopt novel risk modeling and optimization techniques to daily portfolio management to develop and compare specialized methods for scenario generation and scenario tree construction. They also discuss a two-stage stochastic programming problem with conditional measures of risk, which is used to re-balance the portfolio on a rolling horizon basis, with transaction costs included in the model. Their results are supported by an extensive simulation study on real-world data of several versions of the methodology, based on which they conclude that two-stage models outperform single-stage models in terms of long-term performance and also that using high-order risk measures is superior to first-order measures.

3.2 Applications to Option valuation

Applications in this area have been driven by the need to develop solutions in the context of mathematical complexity. In the case of simple derivatives, like a European option, we have the Black-Scholes formula, based on assumptions of geometric Brownian motion for the stock prices and other generalizing features. When exact formulas are not available, different numerical procedures are used for valuing derivatives. MC simulation is one of the preferred and prominent methods for valuing complex derivatives.

MC simulation is generally used for valuing derivatives where the payoff is dependent on the path of the underlying or where there are many underlying variables. The main advantage of MC simulation is that we can use it when the payoff depends on the path followed by S as well as when it depends only on the final value of S. Also, payoffs can occur any number of times during the life of the derivative and any stochastic process can be accommodated. We can extend the procedure to deal with situations where there are many underlying variables. Though MC simulation can be computationally time consuming and is not well suited for valuing American-style options, researchers have developed different ways to adapt the procedure to handle them , using either a least-squares analysis or by parameterizing the early exercise boundary.

Black-Scholes model for valuing an option is found to be imperfect despite its prevalent use in hedging, as it assumes continuous hedging and zero transaction cost. A specific application (Nilakantan and Talwar, 2014) was developed by replacing the continuous hedging assumption with discrete hedging specified number of times a day. Results were obtained by testing the model for European call options on 12 NIFTY-constituent stocks.

The replication strategy was studied by carrying out Monte Carlo simulations of the model and analysing the results for the uncertainty in the replication error.Three scenarios: hedging once, twice, and four times a day were studied with Monte Carlo simulation and the following conclusions were drawn. 1).The replication error (represented by final profit/ loss) is not zero, in the Indian market.The final distribution of replication error i.e. the final profit/loss is not normal, in all the cases, as also verified with the K-S test. However, representing the risk of final profit/loss in terms of a standard deviation of the distribution is quite useful. 2).The standard deviation obtained in simulations for once-a-day hedging is different/ higher in all the options than the theoretical standard deviation (calculated from the options vega). However, the obtained standard deviation reduces approximately for all stocks by $\sqrt{2}$ and 2, when the hedging is performed twice and four times in a given trading day respectively. Thus the proportion of $\sqrt{N}$ is maintained at both twice and four times a day hedging. It was possible for us to verify the relationship between σ_{PandL} at different hedging frequencies which is proportional to $\sqrt{N}$.

3.3 Applications to Simulation-Optimization

Standard optimization methods assume the property of certainty for the parameters and uncertainty is normally dealt with through stochastic programming. Simulation is a viable process that can be coupled with optimization procedures. A simulation- optimization application was developed in the area of operations and logistics (Nilakantan, 2011) for procurement and transportation of coal from different sources to the generation plants with extensions for dealing with uncertainties associated with import delays and power factor variability. While variability and uncertainty dictated the use of nonlinearity and stochastic programming methodology, the optimization was achieved by applying Monte Carlo simulation in a combination of simulation-optimization on the Risksolver platform. A full-fledged simulation-optimization of a power plant logistics system was earlier demonstrated by Yabin Li and Rong Li (2008).

Liu et al. (2013) discuss an application of simulation-optimization for inventory management at Kroger pharmacy chain. Krogers operations research team, in collaboration with faculty from Wright State University, developed a simulation-optimization approach using empirical distributions to model demand. The system was implemented in October 2011 in all Kroger pharmacies in the United States which not only reduced out-of-stocks substantially but also resulted in increased revenue and reduced costs of inventory and labour.

4 SIMULATION ENABLERS

Though simulations can be set up in Excel and other spreadsheets, setting up spreadsheets for complex problems is difficult in many cases for which use of more sophisticated simulation packages is advised. Advanced simulation packages are available to perform the simulation more efficiently and with additional features. Specialized software packages are quite useful to deal with various aspects of the model and provide relevant results from the simulation runs. There are many simulation languages like Arena, GPSS/SLX, SIMPLE++, SIMUL8 etc., which score over general-purpose programming languages in terms of providing a natural framework, lower development cost, simulation-specific error detection etc.

The Systems Modeling Language (SysML) is a general purpose visual modeling language for systems engineering applications. SysML is defined as a dialect of the Unified Modeling Language (UML) standard, and supports the specification, analysis, design, verification and validation of a broad range of systems and systems-of-systems. During the last decade SysML has evolved into enabling technology for Model-Based Systems Engineering (MBSE) for applying rigorous visual modeling principles and best practices to Systems Engineering activities throughout the System Development Life Cycle (SDLC). According to Huang, Ramamurthy, and McGinnis (2007), “Simulation languages and the GUIs supporting them may be excellent tools for creating simulation codes, but are not necessarily the best tools to use for creating descriptions of systems, i.e. for modeling.”After using SysML both to model a system and to support the automatic generation of simulation models, they conclude that “SysML shows great promise for creating object-oriented models of systems that incorporate not only software, but also people, material, and other physical resources, expressing both structure and behavior for such systems.”

QSIM from SAS Inc. is a G-P simulation package used for constructing and analyzing discrete-event simulation models (SAS, 2004). The design of the application makes model building simple for novice users, while a broad array of components and functions makes it appropriate for expert users as well. The ability to create composite components reduces development time when complex model fragments are used over and over.

4.1 Simulation as a Teaching Aid

Management simulations have become prevalent in the education system over the past several decades. While Harvard and other business schools have developed many simulation games to teach management strategies to business school students, we will discuss a few specific applications developed for optimization problem solving and to promote the teaching of simulation in classrooms. Salas, Wildman, and Piccolo (2009) suggest that simulation-based training (SBT) offers many advantages as an approach for management education, and provide several practical guidelines regarding how best to implement simulation-based training in the classroom.

Simulation can be taught with the help of Excel. Paul Jensen (1936-2011) developed Excel-based computational tools to go with Operations Research. His website www.ormm.net was originally created to support the text Operations Research Models and Methods, but it has grown in content much beyond the text. The site is remarkably effective in supporting the goals of OR education and practice. The site provides over 30 add-ins for Microsoft Excel that implement OR methods and includes add-ins for running simulation-optimization models. The testimonials page of the site lists hundreds of comments from students and practitioners attesting to the usefulness of the site and the add-ins. Hopefully, these add-ins are still available even after his passing away. Some of these can still be accessed through his official website at the University of Texas, at Austin.

Snarr and Gold (2006) posit the use of mathematical models and simulation in teaching of macroeconomics at the intermediate level. While models have become complex and sophisticated for students to learn, mathematical software such as Maple has been used to design simulations as pedagogical aids. Based on the design of a system of equations to model the economy, thereby simulating the system with Maple and a pilot test of the simulation, they conclude that symbolic mathematics software can be an effective and student learning tool teaching tool.

Rollins, Gunturi, and Sullivan (2014) discuss, in their paper, a pharma business simulation set up with inputs and hands-on exercises by their students. The design of this practical exercise involved their pharmacy management students who made calculations and global decisions, entered their data into a business simulation software and developed a realistic community pharmacy marketplace. Based on this exercise, their conclusion was that the pharmacy simulation program was an effective active-learning exercise which enhanced students knowledge and understanding of the business.

Liu et al. (2013) developed a spreadsheet model that incorporated a simulation of the ordering process and an iterative procedure to search for near-optimal solutions in a national chain of pharmacies, and is easy to understand by students and practitioners. The spreadsheet model is well- integrated into the curriculum of several engineering courses including inventory management, simulation, and optimization and has provided an interactive environment for students to experience real-life inventory decision making.

5 CONCLUSIONS

In this talk, we have presented a broad overview of the area of Monte Carlo simulation in the context of business applications. A few applications to business were discussed, especially where models are either complex or not analytically tractable. Brief ideas were presented about softwares and teaching aids with which we can solve seemingly complex problems through Monte Carlo simulation. It is hoped that the above discussion will lead to better understanding of the utility of MC simulation to business analysis and decision making as well as its implementation procedures.

REFERENCES

[1] Delta Commissioner of Holland (2013): “the winners of the 2013 Edelman Award, for Economically Efficient Standards to Protect the Netherlands against Flooding”, - https://www.informs.org/About-INFORMS/News-Room/Press-Releases/Edelman-Award-Winner-2013-Dutch-Delta.

[2] Gulten, S. and A. Ruszczynski. (2015): “Two-stage Portfolio Optimization withHigher-Order Conditional Measures of Risk”, Annals of Operations Research, June 2015, Volume 229, Issue1, pp 409-427.

[3] Huang, E., R. Ramamurthy, and L.F. McGinnis. (2007): “System and Simulation Modeling using SysML,” WSC07 Proceedings of the 39th conference on Winter simulation, pp 796-803, ISBN: 1-4244-1306-0.

[4] Kaplan, Paul D. and Sam Savage, (2009): “Markowitz at Mach 1,” Morningstar research paper, December 2009.

[5] Kempf, K.G., et al. (2013): “Optimizing Capital Investment Decisions at Intel Corporation”, - Interfaces, vol 43, issue 1, Permalink: http://dx.doi.org/10.1287/inte.1120.0659,Published Online: February 1, 2013, 62 78.

[6] Liu et al. (2013): “Kroger Uses Simulation-Optimization to Improve Pharmacy Inventory Management”, Interfaces, vol.44, issue 1.

[7] Markowitz, H. (1952): “Portfolio Selection”, the Journal of Finance, Vol. 7, No. 1. (Mar. 1952), pp. 77-91.

[8] Nilakantan, N.S. (2011): “Optimization Modelling For Coal Imports in A Power Plant”, International Journal of Operations Research and Optimization July-December 2011, Volume 2, No. 2, pp-247-260.

[9] Nilakantan, N.S., and S. Talwar. (2014): “Discrete Delta Hedging: Indian Market, SCMS Journal of Indian Management”, Volume XI Number 4, October - December 2014.

[10] Rollins, B.L., et al. (2014): “A Pharmacy Business Management Simulation Exercise as a Practical Application of Business Management Material and Principles”, Am J Pharm Edu, 2014 Apr 17, (78.3), 62.

[11] Salas, E., J.L. Wildman, and R.F. Piccolo. (2009): "Using Simulation-Based Training to Enhance Management Education", Academy of Management Learning and Education, 2009, Vol. 8, No. 4, 559573.

[12] SAS Institute Inc. (2004): "SAS/OR 9.1 Users Guide: QSIM Application", Cary, NC: SAS Institute Inc.

[13] Saucier, R (2000): "Computer Generated Statistical Distributions", Army Research Laboratory, ARL TR-2168.

[14] Savage, S. (2009): "Flaw of Averages", by Sam Savage, John Wiley and Sons, 2009.

[15] Snarr, H. and Steven Gold. (2006): "The Design and use of a macroeconomics simulation using Maple software: A Pilot study" - Accessed from http://scholarworks.rit.edu/other/595.

[16] Yabin Li, Rong Li, (2008),"Simulation and Optimization of the Power Station Coal-Fired Logistics System Based on Witness Simulation Software," -Workshop on Power Electronics and Intelligent Transportation System, 2008, pp. 394-398.